Date Planned : __ / __ / __	Daily Tutorial Sheet - 5	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	JEE Advanced (Archive)	Exact Duration :________

41. Evaluate $\int_0^{\pi} \frac{x dx}{1+\cos\alpha \sin x}, 0<\alpha<\pi$. **(1986)**

42. Evaluate: $\int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x+\sin^4 x} dx$. **(1985)**

43. Evaluate $\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$ **(1984)**

44. Evaluate $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16 \sin 2x} dx$ **(1983)**

45. (i) Show that $\int_0^{\pi} x f(\sin x)\, dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x) dx$. **(1982)**

(ii) Find the value of $\int_{-1}^{3/2} |x \sin \pi x| dx$

46. Evaluate $\int_0^1 (tx+1-x)^n dx$, where n is a positive integer and t is a parameter independent of x. Hence show that $\int_0^1 x^k (1-x)^{n-k} dx = \frac{1}{{}^nC_k (n+1)}$, for all $k=0,1,\ldots n$. **(1981)**

47. The value of $\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}(1-x^2)^5\right\} dx$ is: **(2014)**

48. Let $g(x)=\int_0^x f(t)dt$, where f is such that $\frac{1}{2} \le f(t) \le 1$ for $t \in [0,1]$ *and* $0 \le f(t) \le \frac{1}{2}$ *for* $t \in [1,2]$.

Then, $g(2)$ satisfies the inequality **(2000)**

(A) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ **(B)** $0 \le g(2) < 2$

(C) $\frac{3}{2} < g(2) \le \frac{5}{2}$ **(D)** $2 < g(2) < 4$

49. Show that $\int_0^{n\pi+v} |\sin x| dx = 2n+1-\cos v$, where n is a positive integer and $0 \le v < \pi$. **(1994)**

50. Given a function $f(x)$ such that it is integrable over every interval on the real line and $f(t+x)=f(x)$, for every x and a real t, then show that the integral $\int_a^{a+t} f(x)\, dx$ is independent of a. **(1984)**